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AN ELASTOPLASTIC MODEL FOR UNSATURATED EXPANSIVE SOILS BASED ON SHAKEDOWN CONCEPT

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Abstract. It is important to model the behaviour of unsaturated expansive soils subjected to hydromechanical loadings, because these wetting and drying cycles alter significantly their hydromechanical behaviour which may cause a huge differential settlement on the foundations of individual buildings, pavements, dams, etc. From experimental observations, these expansive soils can generally reach a final equilibrium state at the end of the suction cycles where the soil behaviour can be supposed elastic. In this context, this paper presents an analytical method based on shakedown concept for the hydromechanical behaviour of expansive soils. The required parameters of the shakedown-based model are calibrated by the experimental results obtained for bentonite/sand mixtures subjected to cyclic suction loadings in an oedometric test. The comparison between the experimental results and the modeling demonstrates the capacity of the proposed shakedown-based model to simulate the hydromechanical behaviour of unsaturated expansive soils.

1 INTRODUCTION

The expansive clayey soil is extensively applied to various domains in civil engineering, and its behaviour is influenced by natural environment fluctuations. To investigate the hydromechanical behaviour of expansive soils, it is necessary to determine the properties of these materials from experiments. Laboratory tests on these materials during wetting and drying cycles have been reported by several authors[1, 2, 3, 4, 5, 6, 7]. All these results show that the equilibrium state can be reached where presents the reversible behaviour after several suction cycles. To resolve this problem, it requires a simple constitutive model to predict the hydromechanical behaviour of unsaturated expansive soils during wetting and drying cycles.

Barcelona Expansive Model (BExM) is the most accepted theoretical reference among numerous models, which was presented in 1999 by Alonso et al [8]. It is able to simulate the basic behaviour of unsaturated expansive soil, including the strain fatigue phenomenon during drying-wetting cycles and the prediction of final equilibrium state at the end of the suction cycles. However, it is difficult to determine the coupling functions for micro- and macrostructural strains, and leads to unrealistic calculation time for large number of cycles.

The shakedown concept applied to unbound granular materials was firstly introduced by Sharp et al.[9], which defined shakedown load as the key design load. Zarka simplified the shakedown theory which defines the plastic strains at shakedown states with Melan's static theorem extended to kinematic hardening materials [10]. Habihallah and Chazallon [11] have applied the simplified shakedown theory to unbound granular materials with a non-associated flow rule. But these models have never been adopted to unsaturated expansive soils subjected to cyclic suction loadings.

Therefore, to improve the modeling of unsaturated expansive soils and calculation for cyclic suction loadings, shakedown concept should be considered to model the hydromechanical behaviour of unsaturated expansive soils.

2 ZARKA'S METHOD FOR MECHANICAL LOADINGS

For this simplified method, Zarka introduces the transformed internal variables to characterize the mechanical system, then a local geometrical construction in transformed internal parameter plane is performed to estimate the stabilized limit state and the associated plastic components.

In the case of 1-D deformation for kinematic hardening material, the yield surface equation can be written as:

$$f = |\sigma - y_\alpha| - \sigma_\alpha \tag{1}$$

where, σ_α is the yield stress(also called local stress) and y_α(the kinematic hardening variable) is related to the plastic strain,

$$y_\alpha = h \cdot \varepsilon^p \tag{2}$$

where, h is the kinematic hardening modulus.

We rewrite the local stress at the level of the plastic mechanism (σ_α) in terms of the given applied stress (σ) and the transformed internal variable (y_α):

$$\sigma_\alpha = \sigma - y_\alpha \tag{3}$$

where, y_α presents the transformed internal parameter.

Two cases exist according to the loading amplitude ($\Delta\sigma$) in the transformed internal parameter plane:

• The extreme positions of the convex centered in σ_{min} and σ_{max} have a common part and elastic shakedown will occur (see Figure 1a). In this case, the plastic strain can be calculated by:

$$\Delta\varepsilon^p = \frac{\Delta\sigma}{h} \tag{4}$$

where, $\Delta\sigma$ is the difference of σ_{max} and σ_{min}.

When elastic shakedown occurs, the response of the material becomes purely elastic after a certain number of loading cycles and plastic strains tend towards a constant depending on the initial state of the material. The stress-plastic strains curve is illustrated for elastic shakedown(Figure 1b).

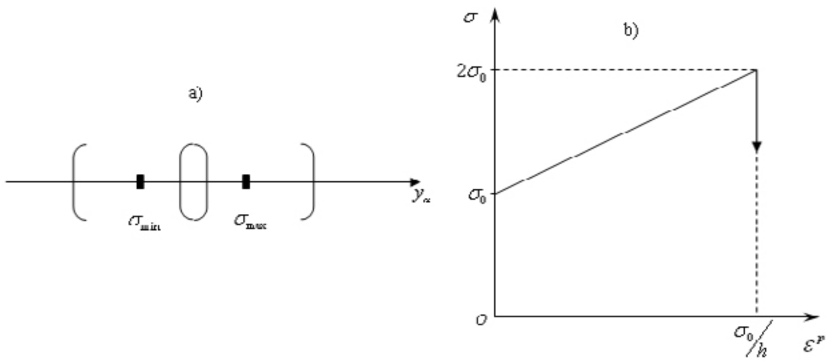

Figure 1: Elastic shakedown: a) transformed internal parameter plane b) stress-plastic strain curve

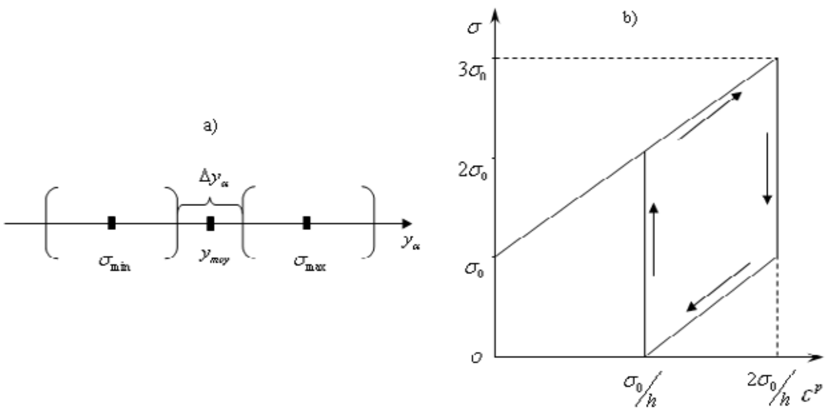

Figure 2: Plastic shakedown: a) transformed internal parameter plane b) stress-plastic strain curve

• The extreme positions of the convex have no common part and in this case, plastic shakedown will happen when loading amplitude ($\Delta\sigma$) is large (Figure 2a). For this, the maximum plastic strain and the minimum plastic strain between σ_{max} and σ_{min} can be defined in the transformed internal parameter plane:

$$\varepsilon^p_{max} = \frac{1}{h} \cdot (y_{moy} + \frac{1}{2} \cdot \Delta y_\alpha) \tag{5}$$

$$\varepsilon^p_{min} = \frac{1}{h} \cdot (y_{moy} - \frac{1}{2} \cdot \Delta y_\alpha) \tag{6}$$

where, y_{moy} presented in figure 2a is equal to $(\sigma_{max} + \sigma_{min})/2$.

In this case, the response of the material becomes periodic during cycles with periodic plastic strain. In Figure 2b, a closed elastic-plastic loop is obtained during plastic shakedown which can be treated as a steady state, with no accumulation of plastic deformation.

3 SHAKEDOWN MODELING FOR HYDROMECHANICAL LOADINGS

It is generally accepted that a unique set of the net mean stress and the total suction are efficient to describe the hydromechanical behaviour of unsaturated soils. The equations of yield surfaces are given by Suction Increase (s_I) limit, Suction Decrease (s_D) limit, and Preconsolidation Stress(p_0),

$$s = s_I \tag{7}$$

$$s = s_D \tag{8}$$

$$p = p_0 \tag{9}$$

where, s_I, s_D and p_0 define the elastic region (see Figure 3).

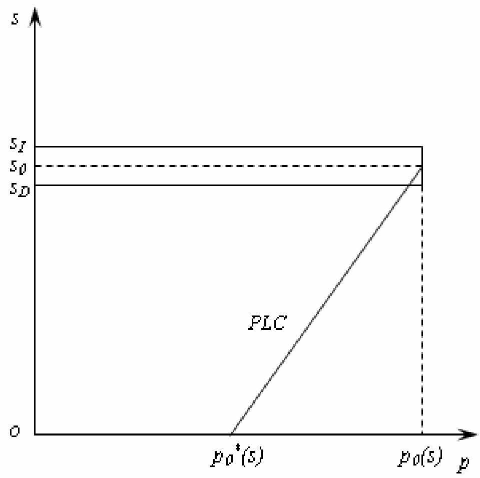

Figure 3: Yield surfaces for simplified model

We define Pseudo Loading Collapse (PLC) with a linear relation between the preconsolidation stress and suction level. This relation can be given by:

$$p_0(s) = A \cdot s + B \tag{10}$$

where, A and B can be identified by the isotropic compression tests at different suctions.

Parameter A is a constant during suction cycles, controlling the slope of PLC yield surface.

Parameter B can be given by,

$$B = p_0^*(\gamma_d) \tag{11}$$

where, $p_0^*(\gamma_d)$ is the preconsolidation stress for saturated state, changing with the initial dry density(initial state).

3.1 Coupled elastoplastic framework

Two types of elastoplastic strains can be developed due to suction variations or mechanical loadings.

1) Elastoplastic strains for suction variations

The suction variation within the yield surface will result in the elastic volumetric strain:

$$d\varepsilon_{vs}^{e} = \frac{\kappa_s}{v} \cdot \frac{ds_0}{s_0} \tag{12}$$

and if the boundaries ($s = s_I$ and $s = s_D$) are activated, the following total and plastic volumetric deformation will be generated,

$$d\varepsilon_{vs} = \frac{\lambda_s}{v} \cdot \frac{ds_0}{s_0} \tag{13}$$

$$d\varepsilon_{vs}^{p} = \frac{\lambda_s - \kappa_s}{v} \cdot \frac{ds_0}{s_0} \tag{14}$$

in which, κ_s and λ_s are elastic stiffness index and elastoplastic stiffness index for suction variation, respectively.

2) Elasoplastic strains for mechanical loadings

Similarly, the increase of stress within the yield surface will generate the elastic volumetric strain:

$$d\varepsilon_{vp}^{e} = \frac{\kappa}{v} \cdot \frac{dp}{p} \tag{15}$$

and if the boundary $p = p_0(s)$ is reached, the following total and plastic volumetric deformation will be given by:

$$d\varepsilon_{vp} = \frac{\lambda(s)}{v} \cdot \frac{dp}{p} \tag{16}$$

$$d\varepsilon_{vp}^{p} = \frac{\lambda(s) - \kappa}{v} \cdot \frac{dp}{p} \tag{17}$$

in which, κ and $\lambda(s)$ are elastic stiffness index and elastoplastic stiffness index for loading variation, respectively.

3.2 Plastic shakedown during suction cycles

When the loading amplitude becomes very large, a stabilized limit state can be reached at the end of suction cycles. The transformed internal parameter (y_α) as well as the preconsolidation stress (p) are presented in the same plane in Figure 4. In the transformed internal parameter plane, the convex that characterizes the behaviour of the soil sample translates between the minimum suction (s_{min}) and the maximum suction (s_{max}) during the drying and wetting cycles. If the extreme positions of the convex have no common part in the transformed internal parameter axis, plastic shakedown will occur. Therefore, the variation of the volumetric plastic deformation ($\Delta\varepsilon_{vs}^{p}$) during suction cycles can be computed by:

$$\Delta\varepsilon_{vs}^{p} = \frac{1}{h} \cdot \Delta y_\alpha \tag{18}$$

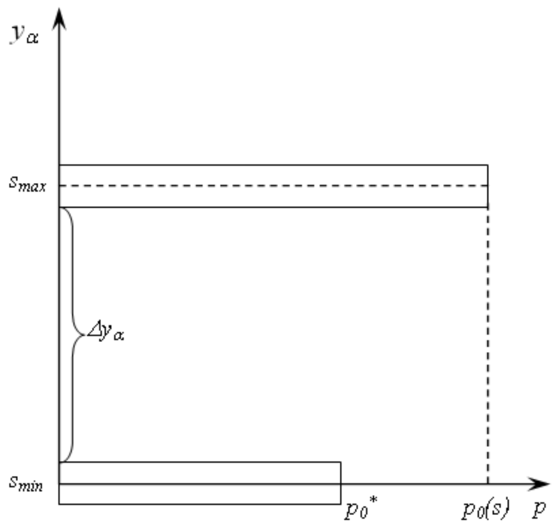

Figure 4: Transformed internal parameter-preconsolidation stress plane for plastic shakedown

3.3 Elastic behaviour at the equilibrium state

After several drying and wetting cycles, a stable state will be obtained at the end of the suction cycles in the volumetric strain-suction plane (see Figure 5) where no plastic strain accumulation can be observed at the final equilibrium state. Consequently, a linear variation of the elastic strain with the suction can be supposed at the equilibrium state. It can be written as:

$$\Delta\varepsilon_{vs}^{e} = \frac{\kappa_s}{v} \cdot \Delta s \tag{19}$$

where, κ_s is the elastic stiffness index for suction variation.

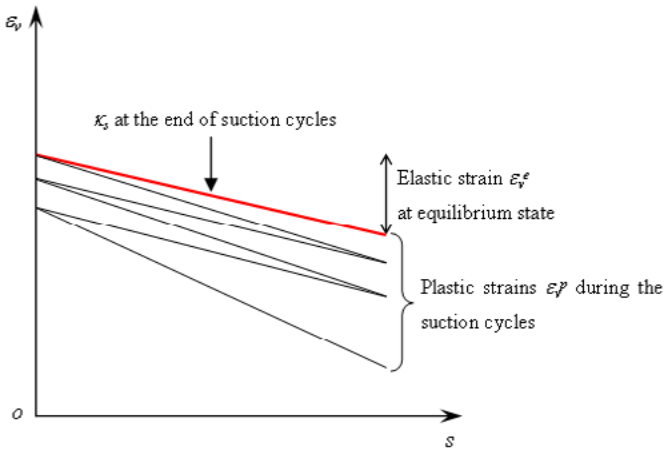

Figure 5: Variation of volumetric strain with suction cycles

4 PARAMETER CALIBRATION FOR SHAKEDOWN-BASED MODEL

The capacity of our modified shakedown concept to model the hydro-mechanical behaviour of the expansive soils is investigated for the experimental results published by Alonso et al.(2005). Three groups of cyclic wetting and drying between 0 and 140 MPa under constant net mean stress of 98, 196 and 396 kPa were performed on a compacted mixture of bentonite-sand with an initial dry density of 1.5 Mg/m^3. The differences between two successive wetting and drying paths become smaller as the number of cycles increases and eventually shrinkage strain is accumulated. Clearly, the soil tends towards an elastic state at the end of suction cycles.

Table 1 summarizes the calibrated parameters for the shakedown-based model for these results. The wetting and drying cycle tests at a given net mean stress can provide the hardening modulus (h) and the elastic stiffness index for suction variation (κ_s), while the width of two elastic boundaries s_I and s_D is very small, taken equal to 0.1 MPa. In addition, κ_s can be considered constant according to the experimental results. Because of the volumetric shrinkage strains during the suction cycles, a positive sign was used for the h values to show the hardening phenomenon.

Table 1: Calibration of required parameters of shakedown-based model for the studied material of Alonso et al.[2]

Parameter	98kPa	196kPa	396kPa
h(MPa)	3784	3111	2000
$s_I - s_D$(MPa)	0.1	0.1	0.1
κ_s	0.001	0.001	0.001

Figure 6 illustrates the evolution of hardening parameter with different net mean stress and a linear fit between the inverse of hardening modulus ($1/h$) and net mean stress was proposed. The larger the applied vertical stress (p), the larger the inverse of the hardening modulus ($1/h$). In other words, the accumulated plastic deformation increases with an increase of the net mean stress.

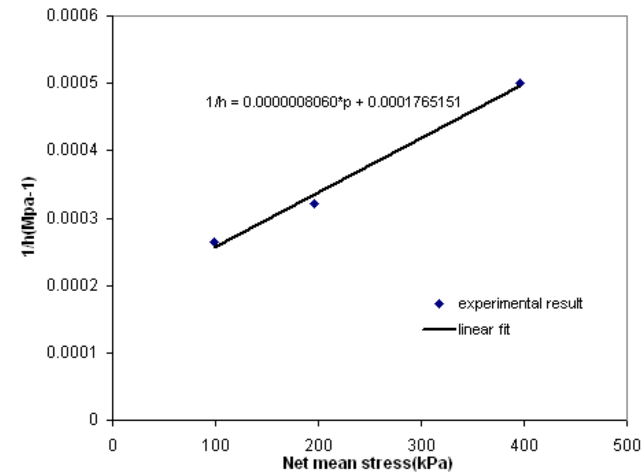

Figure 6: Evolution law of the inverse of the hardening modulus for the samples studied by Alonso et al.[2]

Here, we define the following linear expression between the inverse of the hardening modulus ($1/h$) and the net mean stress (p),

$$1/h = C \cdot p + D \tag{20}$$

where, C and D are material constants. Table 2 shows the variation of these parameters for this studied material.

Table 2: Variations of parameter C and D for studied material of Alonso et al.[2]

Parameter	$C(\mathrm{MPa}^{-2})$	$D(\mathrm{MPa}^{-1})$
Value	0.0008060	0.0001765151

The comparison between test results and the model estimation at a given net mean stress (196 kPa) is presented in Figure 7. It can be generally stated that the shakedown-based model demonstrates a pretty good agreement with the tests.

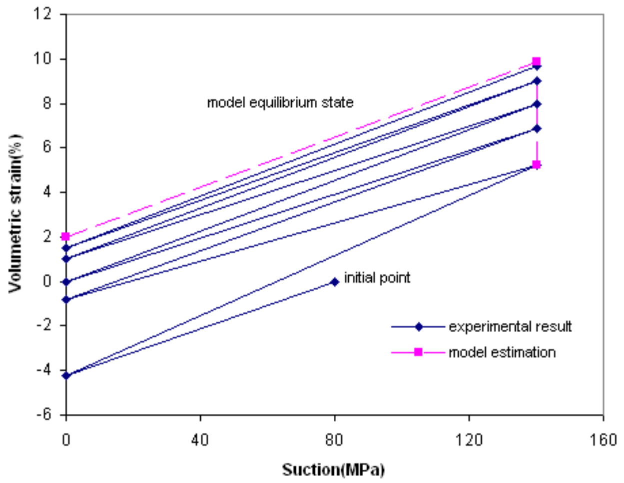

Figure 7: Comparison of test results at the net mean stress of 196kPa reported by Alonso et al.[2] with model estimations

5 CONCLUSIONS

In this paper, the shakedown-based model was proposed to simulate the hydromechanical behaviour of unsaturated expansive soils. The following conclusions are made on the simulations:

- The plastic shakedown was used in the transformed internal parameter plane with a rectangular shape as the yield surface and it happens during the suction cycles when there is no intersection between two extreme positions of the convex;

- Parameter calibrations for shakedown-based model require to determine three parameters: hardening parameter(h), elastic stiffness index for suction variation(κ_s) and elastic region limit(s_I-s_D) from suction cycle tests at a constant net mean stress;

- Necessary comparisons of the laboratory tests with model calculations were performed. It demonstrates that the proposed model is able to simulate the hydromechanical behaviour of unsaturated expansive soils.

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